SET THEORY WORKSHOP
COMPACTNESS AND CARDINAL INVARIANTS
MAY 3, 2024

1. Program

MORNING session 9:00-12:00, Oskar-Morgenstern-Platz 1, SR 6
AFTERNOON session 14:00-17:55, Kolingasse 14-16, SR 10

$09:00-9:30$	Andrea Medini
$09:35-10:05$	Šárka Stejskalová
$10:05-10:40$	COFFEE
$10:40-11:10$	Corey Switzer
$11:15-11:45$	Serhii Bardyla
$12:00-14:00$	LUNCH
$14:00-14:30$	Chris Lambie-Hanson
$14:35-15:05$	Jonathan Cancino
$15:05-15:40$	COFFEE
$15:40-16:10$	Julia Millhous
$16:15-16:45$	Tristan van der Vlugt
$16:50-17:20$	Martina Iannella
$17:25-17:55$	Claudio Agostini

2. Streaming Info: Zoom

Topic: Set Theory Workshop
Time: May 3, 2024 09:00 AM Vienna

Meeting ID: 656 9800 8276
Passcode: kgrc

Organizers:
Vera Fischer (Uni Wien) <vera.fischer@univie.ac.at>
Radek Honzik (Charles University) <radek.honzik@ff.cuni.cz>

If you have any questions, please write to the organizers.

3. Abstracts

Claudio Agostini

Title: On Nagata-Smirnov spaces and metrizability-like properties

Abstract: Metrizable spaces are fundamental in many areas of mathematics, including descriptive set theory. The quest to characterize metrizability has led to significant results, such as Urysohn's metrization theorem, the Nagata-Smirnov-Bing metrization theorem, Arhangel'skij's metrization theorem, and more. These theorems highlight key topological properties that contribute to the desirable behavior of metric spaces. These properties can also be used in generalized descriptive set theory to define new classes of non-first-countable (and thus non-metrizable) topological spaces, extending results from classical descriptive set theory about Polish spaces to these new classes.

In this talk, I will explore classes of topological spaces defined by extending metrizable-like properties to the uncountable setting, like Nagata-Smirnov spaces. This is joint work with Luca Motto Ros.

Serhii Bardyla

Title: Bohr compactification of discrete groups and Schur ultrafilters

Abstract: We give a brief introduction to semigroups of ultrafilters, Bohr compactifications and CHART groups. Then we discuss Schur ultrafilters, and with their help find an alternative description of Bohr compactification of discrete groups. This approach allows us to characterize CHART topological groups. Namely, a CHART group G is a topological group if and only if each Schur ultrafilter on G converges to the unit of G. The talk is planned to be accessible for a broad audience and, as such, no specific knowledge of Algebra or Topology is required.

Jonathan Cancino

Title: Some results on Tukey types of ultrafilters on the natural numbers

Abstract: We will review some constructions regarding Tukey types of ultrafilters on the natural numbers; for example, a construction of a basically generated ultrafilter which is not a nowhere dense ultrafilter.

Martina Iannella

Title: Descriptive consequences of rank-into-rank axioms

Abstract: In this talk I present results contributing to the study of generalisations of the Perfect Set Property and the Baire Property to subsets of spaces of higher cardinalities, such as the power set $\mathcal{P}(\lambda)$ of a singular cardinal λ of countable cofinality. We discuss under which large cardinal hypotheses such properties hold for simply definable subsets of these spaces. This concerns in particular rank-into-rank axioms and sets defined by Σ_1-formulas with parameters from different collections of sets. The presented results stem from joint work with with Vincenzo Dimonte (University of Udine) and Philipp Lücke (University of Hamburg).

Chris Lambie-Hanson

Title: Hajnal-Máté graphs and club guessing

Abstract: We present some results about the existence of certain uncountably chromatic Hajnal-Máté graphs. In particular, we answer a question of Dániel Soukup by proving that there always exists a triangle-free uncountably chromatic Hajnal-Máté graph in the forcing extension by a single Cohen real. Along the way, we isolate some weak club-guessing principles that play a key role in our arguments and seem to be of independent interest. We show that certain of these guessing principles are true in ZFC, while others are independent. This is joint work with Dávid Uhrik.

Andrea Medini

Title: A complete classification of the zero-dimensional homogeneous spaces under determinacy

Abstract: This will be a very informal, proofs-free talk. My main goal will be to introduce and motivate the notions needed to state the classification mentioned in the title. Bonus features (still under AD): a complete classification of the filters on ω up to homeomorphism, and a purely topological characterization of the filters/semifilters on ω. The Borel versions of these results are due in almost all cases to van Engelen.

Julia Millhouse

Title: Projectively definable mad families of multiple sizes

Abstract: I will discuss available strategies and potential difficulties of constructing models of large continuum containing mad families of all possible cardinalities, and moreover each cardinality has an optimal projectively definable witness. In particular I will mention recent progress on such a model where $\mathfrak{c} = \aleph_2$, as well as future plans to adapt this construction to also witness the inequality $\mathfrak{b} < \mathfrak{s}$. This is joint work with Vera Fischer.

Šárka Stejskalová

Title: TBA

Corey Switzer

Title: Baumgartner's axiom and its higher dimensional versions

Abstract: A set of reals $A \subseteq \mathbb{R}$ is called $\aleph_1$-dense if its intersection with every nonempty open interval has size $\aleph_1$. Baumgartner's axiom (BA) is the statement that every pair of $\aleph_1$-dense set of reals are isomorphic as linear orders. BA is the most straightforward generalization of Cantor's theorem about countable dense linear orders to the uncountable. This axiom, proved consistent by Baumgartner in 1973, while seemingly innocuous is actually both very finnicky and also seems to induce a lot of structure on the reals. For instance (on the finnicky side) it is implied by PFA, but not MA, even in the presence of strong reflection principles. On the "induces a lot of structure" side, it implies the bounding number is greater than $\aleph_1$ (Todorčević). BA also has a

natural generalization to higher dimensions i.e. $\mathbb{R}^n$ for $n > 1$ and these versions do follow from MA and in fact weaker cardinal characteristic assumptions (Steprāns-Watson). In this talk we will discuss these issues and show that the higher dimensional versions however also induce a lot of structure on the reals, in particular for every natural number n BA for $\mathbb{R}^n$ implies the bounding number is bigger than $\aleph_1$. This is joint work with Juris Steprāns.

Tristan van der Vlugt

Title: The horizontal direction

Abstract: We will take a look at cardinal characteristics of the Cichoń diagram on the higher (or generalised) Baire space. Consistency results that separate the diagram in a vertical manner are found with relative ease (using higher analogues of the Cohen or Hechler models), but separation in a horizontal manner appears to be quite hard. We take a look at why some of the obvious ideas do not work and present several open problems.