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FAKULTÄT FÜR MATHEMATIK
Dekan Univ.-Prof. Dr. Radu Ioan Bot

Einladung zur öffentlichen Defensio

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Thema der Dissertation
Hyperseries and generalized real analytic functions

Abstract:

In this defense talk, I present some developments of the theory of *generalized real analytic functions* (GRAF) in the setting of Colombeau theory of generalized functions. Since the ring ${}^{\rho}\widetilde{\mathbb{R}}$ of Robinson-Colombeau is non-Archimedean and Cauchy complete, a classical series $\sum_{n=0}^{+\infty} a_n$ of generalized numbers $a_n \in {}^{\rho}\widetilde{\mathbb{R}}$ is convergent if and only if $a_n \to 0$ in the sharp topology, which is the natural topology for these generalized functions. This property does not permit us to generalize several classical results in the study of analytic generalized functions as defined by ordinary series. A natural solution is hence to develop the notion of *hyperseries*, i.e. of series extended over the set ${}^{\rho}\widetilde{\mathbb{N}} \subseteq {}^{\rho}\widetilde{\mathbb{R}}$ of finite and infinite natural numbers. This notion naturally needs the preliminary concept of hyperlimit, i.e. of limit $n \to +\infty$ for $n \in {}^{\rho}\widetilde{\mathbb{N}}$. Both for studying monotone hyperlimits and the radius of convergence of hyper-power series, we had to consider that ${}^{\rho}\widetilde{\mathbb{R}}$ is not order-complete, and we therefore studied the related notions of close supremum and infimum. The corresponding definition of hyperseries allows us to extend numerous classical results which do not hold using classical series in a non-Archimedean framework. These foundations allow us to develop the notion of GRAF by studying hyper-power series and proving classical results such as algebraic operations, composition and inversion of hyper-power series. On the contrary with respect to the classical use of series, we can recover several classical examples in a non-infinitesimal set of convergence (e.g. the exponential, or the Dirac delta - since the identity principle do not hold in this setting) and all Colombeau real analytic functions. We eventually aim to generalize, in a future work, the Cauchy-Kowalevski theorem to a wider class of generalized functions.

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Topic: Theisis Defense D. Tiwari
Time: Jun 28, 2023 14:00 Vienna

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