FAKULTÄT FÜR MATHEMATIK

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Einladung zur öffentlichen Defensio

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Thema der Dissertation

(Asymptotic) Analysis of the Pauli-Poisswell equation in semi-relativistic Quantum Physics

Abstract:

This thesis is in the field of (asymptotic) analysis and mathematical modeling of time-dependent nonlinear partial differential equations (PDE) in relativistic quantum physics.

The self-consistent Pauli-Poisswell equation for 2-spinors is the first order in 1/c (where c is the speed of light) semi-relativistic approximation of the Dirac-Maxwell equation for 4-spinors. It consists of a vector-valued magnetic Schrödinger equation with the additional „Stern Gerlach" term which couples spin and magnetic field via the Pauli matrices. The magnetostatic approximation of Maxwell's equations yields 1+3 Poisson type equations with density and current as source terms for the self-consistent electromagnetic potential. The Pauli-Poisswell equation is a consistent O(1/c) model which keeps both relativistic effects of magnetism and spin which are both absent in the non-relativistic Schrödinger-Poisson equation and inconsistent in the Schrödinger-Maxwell equation.

We deal with the physically meaningful setting of 3 space dimensions and consider the time evolution fromappropriate initial data. Wigner transforms as the appropriate tool for global-in-time semiclassical limits of the scalar non-relativistic Schrödinger-Poisson equation in a mixed state formulation in 3d towards the Vlasov-Poisson equation were established by P.L. Lions & T. Paul and P. Markowich & N.J. Mauser.

We extend these results to the more difficult Pauli-Poisson equation for 2-spinors, a model where the self-consistent magnetic field is dominated by a strong external magnetic field, while the electric field is obtained self-consistently via the Poisson equation. Here, magnetic Lieb-Thirring estimates are crucial. The Pauli-Poisson equation converges towards the magnetic Vlasov-Poisson equation with Lorentz force.We deal with the Pauli-Poisswell equation where we prove local wellposedness and existence of finite energy weak solutions.

Moreover, we present the global-in-time semiclassical limit of the Pauli-Poisswell equation to the Vlasov-Poisswell equation with Lorentz force by the Wigner method: Substantial progress has been achieved using an additional assumption on the initial mixed state that yields the boundedness of the Husimi function. A technical problem of missing regularity due to the nonlinear coupling of the current density and the presence of the magnetic field remains open.

We also use WKB methods to prove the local-in-time semiclassical limit of the Pauli-Poisswell equation to the Euler-Poisswell equation which corresponds to the Vlasov-Poisswell equation with monokinetic initial data. A key step is to obtain a priori energy estimates for which we have to take into account the Poisson equations for the electromagnetic potentials. We also obtain weak convergence of the monokinetic Wigner transform and strong convergence of the density and the current density.

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