



universität
wien

Fakultät für Mathematik

Mathematisches Kolloquium

Mittwoch, 7. Mai 2025

Sky Lounge

EINLADUNG

Krishnaswami Alladi

(University of Florida)

„Duality between prime factors and primes in arithmetic progressions“

Abstract:

In 1977, I noticed two duality identities connecting the smallest and largest prime factors of integers, and vice-versa, the connection being provided by the Moebius function. Using this duality, I generalized the famous results of Edmund Landau on the Moebius function which are equivalent to the Prime Number Theorem. When this duality is combined with the Prime Number Theorem for Arithmetic Progressions, this leads to the striking result that

$$(1) \quad \sum_{n \geq 2, p(n) \equiv \ell \pmod{k}} \frac{\mu(n)}{n} = -\frac{1}{\phi(k)}$$

for all positive integers k , where $1 \leq \ell \leq k$, $(\ell, k) = 1$, $\mu(n)$ is the Moebius function, $p(n)$ is the smallest prime factor of n , and $\Phi(k)$ is the Euler function. In the last few years, this duality and identity (1) have attracted considerable attention, and extended to the setting of algebraic number fields by the use of the Chebotarev Density Theorem by several young researchers.

In the 1977 work, I established four general duality identities connecting the smallest prime factor with the k -th largest prime factor, and the k -th largest prime factor with the smallest prime factor, utilizing not just $\mu(n)$, but also $\omega(n)$, the number the distinct prime factors of n . Recently, along with my PhD student Jason Johnson, I used the second order duality together with the Prime Number Theorem for Arithmetic Progressions to establish

$$(2) \quad \sum_{n \geq 2, p(n) \equiv \ell \pmod{k}} \frac{\mu(n)\omega(n)}{n} = 0.$$

The proof involves a variety of elementary and analytic techniques, and a study of the distribution of the second largest prime factor of n . Identity (2) has more recently been extended to the setting of algebraic number fields using the Chebotarev Density of Theorem by Sroyon Sengupta, another of my PhD students. Starting from the 1977 results, I will discuss all the recent developments pertaining to this problem.

14.45 Uhr: Kaffeejause

15.15 Uhr: Vortrag

vinum cum pane im Anschluss

Michael Schlosser
Nicolas Smoot
Radu Ioan Boț