



universität  
wien

Fakultät für Mathematik

## EINLADUNG

zu

einem didaktischen Vortrag

im Rahmen der Habilitation von

**Corey Bacal Switzer**

(Universität Wien)

**“THINKING RATIONALLY: WHAT THE RATIONAL NUMBERS  
CAN TELL US ABOUT LOGIC, REAL ANALYSIS AND MORE”**

## **“THINKING RATIONALLY: WHAT THE RATIONAL NUMBERS CAN TELL US ABOUT LOGIC, REAL ANALYSIS AND MORE”**

### Abstract:

Consider the language  $L$  with a single binary relation symbol  $<$ . A linear order,  $A = \langle A, <_A \rangle$  is an  $L$ -structure so that  $<_A$  orders  $A$  linearly i.e.  $<_A$  is a transitive, antireflexive relation so that for all  $x$  and  $y$  we have  $x <_A y$  or  $y <_A x$  or  $x = y$ . Linear orders abound across mathematics - in particular  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$  and  $\mathbb{R}$  are all first understood as linear orders. Amongst these, the rational order  $\mathbb{Q}$  is perhaps the most special. Every countable dense linear order without endpoints is isomorphic to  $\mathbb{Q}$  (a theorem due to Cantor). Even more striking every countable linear order embeds into  $\mathbb{Q}$ . The goal of the lecture is first to explore these ideas a little further, proving Cantor's theorem and the generalization listed above and use them to conclude some interesting consequences in logic, particularly about definability and second, look at how these logical considerations can be applied to other areas of mathematics - particularly real analysis and topology.

This lecture can be thought of as part of the end of the course 250046-1 Mathematische Logik offered in year 2 or 3 of the bachelor program. I am assuming knowledge of the first three quarters or so of that course - namely  $L$ -structures, elementary equivalence, first order definability, the definitions of countable versus uncountable and the fact that  $\mathbb{Q}$  is countable. However, the main results require very little background knowledge and indeed the majority of the lecture should be understandable to anyone with a first course in real analysis.

**Freitag, 22. März 2024,  
15:00 Uhr bis 15:45 Uhr  
Ort: Seminarraum 03,  
1 OG., OMP 1**

Henk Bruin  
Radu Bot