universität wien

FAKULTÄT FÜR MATHEMATIK
Dekan Univ.-Prof. Dr. Radu Ioan Boţ

Einladung zur öffentlichen Defensio

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Thema der Dissertation

Signature of càdlàg rough paths: universal properties and applications in finance

Abstract:
The theory of rough paths was introduced in Lyons (1998) as a way to understand and overcome the lack of continuity of the Itô-map, which describes the dependence of the solution of a stochastic differential equation (SDE) on the driving signal. Lyons observed that by enhancing the driving process with a finite number of higher-order objects, one can give a meaning in a *pathwise* sense to differential equations driven by *continuous* rough signals, such as Brownian motion. These enhanced or lifted paths are called rough paths. Furthermore, by equipping the space of rough paths with certain variation metrics, the Itô-map becomes continuous. These seminal ideas found many applications, in particular in stochastic analysis, providing a pathwise perspective and stability results to the theory of SDEs.

One important object in this theory is the *signature of a path*, whose original idea goes back to Chen (1957, 1977) but later on, it has been taken up in the context of rough paths and introduced for more general *rougher* signals. The signature can be viewed as an enhancement of a (rough) path through iterated integrals. Its importance is particularly due to its role as a well-suited feature map capturing essential path characteristics. Indeed, invarious respects, linear functionals of the signature act similarly to polynomials on path space and can be viewed as a canonical selection of basis functions. This explains why these signature based methods become more and more popular in machine learning, econometrics and mathematical finance.

While in its early development, the theory of rough paths primarily focused on continuous paths, advancements by Friz and Shekhar (2017) have extended this framework to include càdlàg paths and introduced a notion of signature that encompasses also paths with jumps.

Building on this definition, one main goal of this thesis is to rigorously establish the role of the signature as a linear regression basis for path functionals in the càdlàg setting and to investigate the scope of its applicability from both theoretical and practical perspectives. As a first step we prove a *universal approximation theorem* for continuous (with respect to the J_1-Skorokhod topology) *non-anticipative path functionals* in terms of linear functionals of the signature of càdlàg rough paths.

Then, as an important application, we define a new class of universal signature models based on an augmented Lévy process, which we call *Lévy-type signature models*. The approach that we follow consists in parameterizing the model itself or its semimartingale characteristics as linear functionals of the signature of an augmented Lévy process.

We show that these models extend the continuous signature models for asset prices as proposed so far, while still preserving universality and tractability properties.

Leveraging the universal approximation properties of the signature process, in the second part of the thesis we broaden the scope and introduce a more general class of jump-diffusions, which we call *Sig-jump-diffusions*, whose defining property is that the process semimartingale characteristics are parameterized by linear combinations of potentially infinitely many components of their own signature. For these processes, which includes the class of so-called *holomorphic jump-diffusions* introduced in the thesis, a significant extension of polynomial processes, we show that the expected signature, as well as the expected value of holomorphic functions of the process' marginals, can be computed via duality methods for stochastic processes. In particular, these quantities admit ananalytic representation in terms of a solution of an (extended tensor algebra or sequence-valued) infinite-dimensional linear differential equation.

In the last part of the thesis, we derive a functional Itô-formula for non-anticipative maps of rough paths building on the approximating properties of the signature and as a byproduct of this result we show that sufficiently regular non-anticipative path functionals admit a functional Taylor expansion.

Prüfungssenat

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Zeit und Ort

Freitag, 20. September 2024, 10:30 Uhr

Hybrid:

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Online:

https://univienna.zoom.us/j/66835936280?pwd=82ogNjbJPOsA8MzQyK8qGHtza4352A.1

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