2022 WORLD LOGIC DAY AT THE KURT GÖDEL RESEARCH CENTER

To celebrate the 2022 World Logic Day, the Kurt Gödel Research Center will host two special events: *On January 11th*, the Set Theory Research Seminar will host four talks given by graduate students. Lukas Koschat, Valentin Haberl and Fabian Kaak will speak on topics related to their Master theses. Michal Godzisewski and Lorenzo Sauras will present results from their doctoral dissertation. Schedule and Abstracts are given below. *On January 13th* the Logic Colloquium will welcome Dr. Victor Torres-Perez from the Technical University of Vienna as an invited speaker, who will talk about "Worlds without Martin's Axiom".

Both events will be held on-line. If interested contact *vera.fischer@univie.ac.at*.

Schedule, January 11th

15:00 - 15:30 Lukas Koschat (University of Vienna)

15:45 - 16:15 Valentin Haberl (University of Vienna & Technical University of Graz)

16:30 - 17:00 Fabian Kaak (University of Vienna)

17:15 - 17:45 Michal Godzisewski (University of Warsaw & University of Łódź)

18:00 - 18:30 Lorenzo Sauras (Technical University of Vienna)

Titles & Abstracts, January 11th

Lukas Koschat (University of Vienna)
Title: A short introduction to countable support iterations and proper forcing
Abstract: In this talk I want to present the basic ideas behind countable support iterations and explain why we care about properness. I will give an overview of the fundamental theorems about properness and countable support iterations. In the end I will explain how these tools come together in a proof of the consistency of certain cardinal invariants constellations. The talk is aimed at anyone who is familiar with forcing but not yet familiar with countable support iterations and properness. The content of the talk is mainly extracted from my master thesis, and hence will be accessible to anyone who has completed an introductory course on forcing.

Date: January 6, 2022.

Valentin Haberl (University of Vienna & Technical University of Graz)

Title: Tukey order on hyperspaces of compact subspaces and the Menger covering property

Abstract: For some topological space X the hyperspace $\mathcal{K}(X)$ is the space of compact subspaces of X equipped with the Vietoris topology. By considering the Tukey order for the poset $\mathcal{K}(X)$ ordered by inclusion, it is possible to characterize topological properties of X. For separable metrizable spaces there are the important results $\mathcal{K}(X) \leq_T \omega^\omega$ iff X is Polish and $\mathcal{K}(\mathbb{Q}) \leq_T \mathcal{K}(X)$ iff X is not hereditarily Baire. We look at the characterization $\mathcal{K}(X)$ hereditarily Baire iff X is co-Menger in the special case for subsets of 2^ω. As an application we are able to prove the existence of $X \subseteq 2^\omega$ in ZFC such that $\mathcal{K}(X) \nleq_T \omega^\omega$ and $\mathcal{K}(X) \ngeq_T \mathcal{K}(\mathbb{Q})$. Furthermore, we discuss Menger subspaces in the context of forcing, for example proving that there are $\mathfrak{c}$ many Menger subspaces of 2^ω in the iterated Sacks model.

Fabian Kaak (University of Vienna)

Title:Forcing Indestructibility of MAD families

Abstract: An infinite family $\mathcal{A}$ of infinite subsets of natural numbers is called almost disjoint if any two members of it have finite intersection, furthermore we call it a maximal almost disjoint (short: MAD) family if it is maximal with respect to inclusion. After forcing a MAD family will stay almost disjoint, but the maximality could be destroyed. For a forcing notion $\mathbb{P}$ we say that $\mathcal{A}$ is $\mathbb{P}$-indestructible if it stays maximal in every forcing extension via $\mathbb{P}$.

In this talk I will present a property for forcings, which gives rise to a combinatorial characterization of indestructibility. Using this we can proof implications between indestructibility for different forcing notions. For example if a MAD family is indestructible for some forcing adding a real, then it is Sacks indestructible. The main focus will be the construction of a Sacks indestructible MAD family.

Michał Tomasz Godziszewski (University of Warsaw & University of Łódź)

Title: Spectra of maximal almost orthogonal families of projections in the Calkin algebra

Abstract: Let H be an infinite dimensional separable complex Hilbert space with inner product $\langle \cdot | \cdot \rangle$. Let $\mathcal{B}(H)$ be a Banach space of bounded linear operators on H with the operator norm. In case when $H = \ell^2(\omega)$, we can distinguish a particular subalgebra of the Banach space $\mathcal{B}(H)$: we define $\mathcal{K}(H)$ as the smallest Banach subalgebra of $\mathcal{B}(H)$ containing all finite-dimensional operators, and we call its elements compact operators. So, $T \in \mathcal{B}(H)$ is compact if it is a limit of finite-rank operators. The collection $\mathcal{K}(H)$ has the structure of a C*-algebra and is a ring-theoretical ideal in $\mathcal{B}(H)$.

The Calkin algebra is the quotient C*- algebra $\mathcal{C}(H) = \mathcal{B}(H)/\mathcal{K}(H)$, where the quotient mapping is denoted by $\pi : \mathcal{B}(H) \to \mathcal{C}(H)$. Every separable C*-algebra is isomorphic to a C*-subalgebra of the Calkin algebra. We are interested in the set of projections in the Calkin algebra, i.e., in the set: $P(\mathcal{C}(H)) = \{p \in \mathcal{C}(H) : p = p^* = p^2\}$. For a set $A \subseteq \omega$, let P_A be the projection onto $\ell^2(A) \subseteq \ell^2(\omega)$. The map $A \mapsto P_A$ embeds the Boolean algebra $\mathcal{P}(\omega)$ into the space of projections $P(H)$. The map $A \mapsto \pi(P_A)$ defines an embedding of $\mathcal{P}(\omega)/\operatorname{fin}$ into $P(\mathcal{C}(H))$. This map is called the diagonal embedding.

A family of projections $A \subseteq P(\mathcal{C}(H))$ is almost orthogonal if the product of any two elements $p, q \in A$ is the zero of the algebra $\mathcal{C}(H)$. In this talk we will discuss the possible spectra of maximal almost orthogonal families of projections in the Calkin algebra.

<u>*Lorenzo Sauras*</u> (Technical University of Vienna)

Title: Generalization of proofs of universal sentences

Abstract: In 1999, Baaz established a systematic way of generalizing proofs of universal sentences that has led to certain progress on the challenging problem of the factorization of Fermat numbers (i.e., numbers of the form $2^{2^n} + 1$, where n is a natural number). This talk will explain, in a concise way, how such generalization algorithm works, by inputting an ingenious calculation (i.e., a proof of a quantifier-free formula without variables) of 641 divides $2^{2^5} + 1$ that was discovered by Bennet and Kraïtchik. In addition, if time permits, some other of its outputs (remarkably, sufficient conditions for a given value to be a divisor of a Fermat number), as well as related results from an ongoing research, will be stated.

Logic Colloquium, January 13th

<u>*Victor Torres-Perez*</u> (Technical University of Vienna) will speak at 11:30 (please note unusual time).

Title: Worlds without Martin's Axiom

Abstract: The first of Hilbert's famous list of problems at the beginning of the 20th century was to establish Cantor's Continuum Hypothesis (CH), i.e. if there is no uncountable subset of the reals with cardinality strictly less than the continuum. After the works of Gödel and Cohen, it was concluded that the traditional axioms of Set Theory (ZFC) cannot decide CH.

Since then, new axioms have emerged. Prominently we have Forcing Axioms. One of the first Forcing Axioms ever considered was Martin's Axiom (MA). While MA implies the negation of the CH, it does not decide the exact value of the continuum. However, generalizations of MA like the Proper Forcing Axiom (PFA) or Martin's Maximum (MM) do imply that the continuum is the second uncountable cardinal. Besides, PFA or MM imply the negation of certain square principles or tree properties (among a very large number of interesting consequences). This means in particular that these axioms require the existence of large cardinals.

There are other relatively new principles, which have strong consequences similar to the ones from PFA or MM, but they can coexist consistently with the absence of MA or even imply this absence. A couple of these principles are, for example, Rado's Conjecture (RC) and the P-Ideal Dichotomy (PID). We will give a general review of results involving these kinds of principles, including some of ours obtained along the previous years. There, it is possible to observe that even if they can avoid MA, they are still quite powerful like these traditional Forcing Axioms. We will expose one of our last results, where we prove (with L. Wu) that PID implies the negation of a certain type of two-cardinals square principle.