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**FAKULTÄT FÜR MATHEMATIK**
Dekan Univ.-Prof. Dr. Radu Ioan Boţ

Einladung zur öffentlichen Defensio

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Thema der Dissertation

# Aspects of volatility modeling: from Gaussian processes to martingales with restricted support


Abstract:

This thesis examines two distinct lines of research motivated by volatility modeling. The first part presents a characterization result for finite-dimensional self-similar Gaussian Markov processes, supported by our development of a theory on real matrix semigroups. The second part focuses on the existence of martingale measures with restricted support. The theory developed therein is then applied to formulate no-arbitrage conditions in volatility modeling.

Self-similar Gaussian Markov processes
We characterize all multi-dimensional real self-similar Gaussian Markov processes. Three types of covariance matrix functions occur: white-noise type functions, covariances that can be expressed by continuous matrix semigroups, and covariances based on discontinuous solutions of Cauchy's functional equation. Characterizing the latter requires us to develop some results on the representation theory of non-continuous matrix semigroups, satisfying a mild boundedness assumption, without assuming continuity. In addition to the continuous solutions of the semigroup functional equation, we give a description of solutions arising from non-measurable solutions of Cauchy's functional equation. In dimension one, besides white noise, the self-similar Gaussian Markov processes reduce to a two-parameter family of time-changed Brownian motions. This observation simplifies several proofs of non-Markovianity found in the literature.

Martingales with restricted support
In discrete time financial markets, a sufficient condition for the absence of arbitrage is the existence of a martingale pricing measure that satisfies market-imposed constraints. From

a model-free perspective, this leads to investigating the existence of martingale couplings with restricted domains. However, conventional convex ordering requirements on marginal distributions are insufficient to guarantee their existence. To address this, we reformulate Strassen's theorem, moving away from the convex envelope approach to one that aligns with the geometry of the prescribed domain. This new formulation gives rise to duality results applicable to robust pricing with fixed marginal constraints. In particular we apply these results to investigate the existence of martingale measures compatible with call options and VIX futures.

**Prüfungssenat**

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**Zeit und Ort**

Donnerstag, 17. Oktober 2024, 15:00 Uhr

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