Random planar trees and the Jacobian conjecture

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Abstract

A map $F(x) = (F_1(x_1, \dots, x_n), \dots, F_n(x_1, \dots, x_n))$ from $\mathbb{C}^n$ to itself is called a *polynomial map* if each F_i is a polynomial with complex coefficients in the variables $x_1, \dots, x_n$. In 1939, Keller conjectured that every polynomial map F whose Jacobian determinant is a nonzero constant has a compositional inverse F^{-1} that is itself a polynomial map. This is still one of the greatest unsolved problems of mathematics, and is known as the *Jacobian conjecture*. The combinatorial approach to the Jacobian conjecture, developed more recently from the original context of algebraic geometry, is based on rephrasing the underlying concepts in terms of families of trees. We provide some new insights in this line of research, by using probabilistic methods, such as random trees, Markov chains, and branching processes. This approach also allows us to show that the high degree coefficients of F^{-1} are 'small', thereby proving an approximate version of the Jacobian conjecture.

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