KGRC SET THEORY RESEARCH SEMINAR

The *Set Theory Research Seminar* on December 14th, 2021 will feature **four** talks given by graduate students.

Julia Millhouse and Lukas Schembecker are first year doctoral students at the Kurt Gödel Research Center. They will each give **short** presentations on selected topics (see abstracts below). Marlene Koelbing is in the last semester of her doctoral studies at the University of Vienna, working under the supervision of Professor Sy David Friedman, and Yuxin Zhou is a graduating doctoral student of Professor Jindřich Zapletal at the University of Florida. They will both present results from their **dissertations**.

Schedule

15:00 - 15:25 Julia Millhouse (Uni Wien)
Title: Ramsey uniformization and madness

15:30 - 15:55 Lukas Schembecker (Uni Wien)
Title: Independence of the Whitehead Problem

16:00 - 16:50 Marlene Koelbing (Uni Wien)
Title: Special Aronszajn trees and Kurepa trees

17:00 - 17:50 Yuxin Zhou (University of Florida)
Title: Distinguish chromatic numbers for isosceles triangles in choiceless set theory

Abstracts

Julia Millhouse

Title: Ramsey uniformization and madness

Abstract: A classical result of Mathias states that there do not exist analytic infinite maximal almost disjoint (mad) families. Moreover he proves that all infinite $\mathcal{A} \subseteq [\omega]^\omega$ in Solovay's model satisfy the Ramsey Property; Silver established the same property for analytic sets. Mathias posed the natural question if there exist infinite mad families in Solovay's model, to which Asger Tornquist responded negatively in 2014.

A stronger result appeared in 2019, when Törnquist and Schrittesser proved that if all infinite $\mathcal{A} \subseteq [\omega]^\omega$ have the Ramsey property, then there do not exist infinite mad families. They work in ZF with the fragment of choice, DC. Their proof relies significantly on the assumption of a certain uniformization property, as well as the topological properties of

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a rather canonical object — the equivalence relation of "eventual agreement" on binary sequences. In this talk I will give an outline of this 2019 proof.

<u>*Lukas Schembecker*</u>

Title: Independence of the Whitehead Problem

Abstract: An abelian group A is called Whitehead iff for every abelian group B we have that every short exact sequence of the form

$$0 \to \mathbb{Z} \xrightarrow{\iota} B \xrightarrow{\pi} A \to 0$$

splits, which means that there is a group homomorphism $\rho : A \to B$ such that $\pi \circ \rho = \mathrm{id}_A$, i.e. it satisfies one of the equivalent conditions of the splitting lemma. Clearly, every free abelian group is Whitehead - conversely, we are interested in the Whitehead Problem: Is every Whitehead group free? We will see that for countable groups this is a theorem of ZFC. However, Shelah established the independence of the Whitehead Problem for groups of size $\aleph_1$ - a surprising result as it was the first purely algebraic statement proven to be independent from ZFC. More specifically, we show that the diamond principles in L give a positive answer and MA + $\neg$CH gives a negative answer to the Whitehead Problem.

<u>*Marlene Koelbing*</u>

Title: Special Aronszajn trees and Kurepa trees

Abstract: I will talk about special $\aleph_n$-Aronszajn trees and $\aleph_n$-Kurepa trees. The main result I want to present is the consistency of the statement that the following holds for every $0 < n < \omega$: all $\aleph_n$-Aronszajn trees are special, there are such, and there exists no $\aleph_n$-Kurepa tree. The proof needs ω-many supercompact cardinals. I will discuss the main ideas of the proof.

This is joint work with Sy-David Friedman.

<u>*Yuxin Zhou*</u> (University of Florida)

Title: Distinguish chromatic numbers for isosceles triangles in choiceless set theory

Abstract: For n a positive natural number, let Γ_n be the hypergraph of isosceles triangles on $\mathbb{R}_n$. Under the axiom of choice, the existence of a countable coloring for Γ_n holds for every n. Without the axiom of choice, the chromatic numbers may or may not be countable. With an inaccessible cardinal assumption, there is a model of $\mathsf{ZF} + \mathsf{DC}$ in which Γ_2 has countable chromatic number while Γ_3 has uncountable chromatic number. This result is obtained by a balanced forcing over the symmetric Solovay model.