Tame fields and beyond, IV

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What is wrong with $\mathbb{F}_p((t))$?

We will now describe the original reason for the introduction of the classification of defects. In the first lecture we have already mentioned the following result:

Theorem ([K 2001])

The axiom system

$$\left.\begin{array}{l}\text{“henselian defectless field of characteristic } p \text{ whose} \\ \text{value group is a } \mathbb{Z}\text{-group, and whose residue field is } \mathbb{F}_p\text{”}\end{array}\right\} \quad (1)$$

is not complete.

So what is it that we are missing?

What is wrong with $\mathbb{F}_p((t))$?

We note that for $K = \mathbb{F}_p((t))$, the elements $1, t, t^2, \ldots, t^{p-1}$ form a basis of the field extension $K|K^p$. Thus,

$$K \;=\; K^p \oplus tK^p \oplus \ldots \oplus t^{p-1}K^p \,. \tag{2}$$

Let $\mathcal{L}$ be a language of valued rings or fields. Then the above equation can be expressed by the $\mathcal{L}(t)$-sentence

$$\forall X \exists X_0 \ldots \exists X_{p-1} \;:\; X = X_0^p + tX_1^p + \ldots + t^{p-1}X_{p-1}^p \,. \tag{3}$$

Additive polynomials

A polynomial $f(X) \in K[X]$ is called additive if $f(a+b) = f(a) + f(b)$ for all a, b in any extension field of K. The additive polynomials in $K[X]$ are precisely the polynomials of the form

$$\sum_{i=0}^{m} c_i X^{p^i} \quad \text{with } c_i \in K\,,\ m \in \mathbb{N}\,.$$

For every i the polynomial $t^i X^p$ is additive. Now it is a natural question to ask what might happen if we replace the polynomials $t^i X^p$ in (3) by other additive polynomials. Apart from the additive polynomials cX^{p^n}, the most important for us is the Artin-Schreier polynomial $\wp(X) := X^p - X$.

What is wrong with $\mathbb{F}_p((t))$?

Lou van den Dries observed that if k is a field of characteristic p such that $\wp(k) := \{\wp(x) \mid x \in k\} = k$, then the $\mathcal{L}(t)$-sentence

$$\forall X \exists X_0 \ldots \exists X_{p-1} \;\; X = X_0^p - X_0 + tX_1^p + \ldots + t^{p-1} X_{p-1}^p \tag{4}$$

holds in $k((t))$. However, he was not able to deduce this assertion from axiom system (1) where "$Kv = \mathbb{F}_p$" is replaced by "Kv is perfect and $\wp(Kv) = Kv$". Observe that $\wp(\mathbb{F}_p) = \{0\} \neq \mathbb{F}_p$. To obtain an assertion valid in $\mathbb{F}_p((t))$, we have to introduce a corrective summand Y:

$$\left.\begin{array}{l} \forall X \exists Y \exists X_0 \ldots \exists X_{p-1} \; : \\ X = Y + X_0^p - X_0 + tX_1^p + \ldots + t^{p-1} X_{p-1}^p \;\wedge\; vY \geq 0\,. \end{array}\right\} \tag{5}$$

An example

The following theorem proven in [K 2001] shows that assertion (5) does not follow from axiom system (1):

Theorem

Take (K, v) to be $(\mathbb{F}_p(t), v_t)^h$ or $(\mathbb{F}_p((t)), v_t)$. Then there exists an extension (L, v) of (K, v) such that:
a) $L|K$ is a regular extension of transcendence degree 1,
b) (L, v) satisfies axiom system (1),
c) sentence (5) does not hold in (L, v).

That $L|K$ is a regular extension means that it is separable (i.e., linearly disjoint from the perfect hull of K) and that K is relatively algebraically closed in L.

Construction of the extension

(L, v) is constructed as an algebraic extension of the rational function field $(K(x), v)$, where v is the composition of the x-adic valuation v_x with the t-adic valuation v_t on $K = Kv_x$. Then $vK(x)$ is the lexicographic product $\mathbb{Z} \times \mathbb{Z}$. In order that (L, v) satisfies (1), we must have that $vL/\mathbb{Z} \simeq \mathbb{Q}$, so we have to adjoin suitable radicals. Integrating a twist into this process, we achieve that (5) does not hold in (L, v). However, (L, v) also needs to be henselian and defectless. By going to a maximal immediate algebraic extension, we can achieve that the valued field we are constructing is algebraically maximal. But this does not imply that it is defectless (see Example 3.25 of [K 2011]). So how can we achieve this?

Criterion for henselian defectless fields

We need a criterion for an algebraically maximal field to be a defectless field, that can be realized in constructions. We call a valued field an inseparably defectless field if all of its finite purely inseparable extensions are defectless, and a separable-algebraically maximal field if it admits no nontrivial immediate separable-algebraic extensions. Note that a separable-algebraically maximal field is henselian.

Theorem ([K 2010a])

A valued field of positive characteristic is henselian and defectless if and only if it is separable-algebraically maximal and inseparably defectless.

The proof uses the classification of defect: inseparably defectless fields do not admit extensions with dependent defect.

Ingredients of the proof of the theorem

The proof is pieced together from the following facts:

Proposition

1) If (K, v) is a separable-algebraically maximal field and $(L|K, v)$ is a finite defectless extension, then (L, v) does not admit Artin-Schreier extensions with independent defect.
2) Every finite extension of an inseparably defectless field is again an inseparably defectless field, and thus does not admit Artin-Schreier extensions with dependent defect.

Characterization of inseparably defectless field

We need a criterion for a valued field to be an inseparably defectless field, that can be realized in constructions. The following result is due to F. Delon ([D 1982]):

Proposition

A valued field (K, v) of characteristic $p > 0$ and finite p-degree is inseparably defectless if and only if $(K|K^p, v)$ is a defectless extension, i.e.,

$$[K : K^p] \;=\; (vK : pvK)[Kv : Kv^p] \,.$$

In our construction, the residue field remains $\mathbb{F}_p$, and once we get the value group to be a $\mathbb{Z}$-group, we only have to make sure that the field we construct has p-degree 1. This concludes the construction.

The mixed characteristic case

In mixed characteristic, we do not have an analogous criterion for algebraically maximal fields to be defectless fields. In [K 2010a], the classification of defects was only introduced for valued fields of positive characteristic. There, it was shown:

Proposition

An Artin-Schreier extension $(L|K,v)$ with Artin-Schreier generator ϑ has independent defect if and only if

$$\mathrm{dist}\,(\vartheta,K) \;=\; H^-$$

for some proper convex subgroup H of $\widetilde{vK}$.

Can this in some way serve to generalize the classification to Kummer extensions in mixed characteristic?

The mixed characteristic case

If (K, v) is a valued field of mixed characteristic $(0, p)$ containing all p-th roots and $(L|K, v)$ is a Galois extension of degree p, then it is generated by a Kummer generator η such that $\eta^p \in K$. If the extension has nontrivial defect, then it is immediate and it can be shown that η can be chosen to be a 1-unit, i.e., $v(\eta - 1) > 0$.

In joint work with Anna Rzepka, our first approach was to use the following transformation. Choose $c \in \tilde{K}$ such that $c^{p-1} = -p$, then substitute $X = cY + 1$ in $X^p - \eta^p$ and divide by c^p; this results in a polynomial that is an Artin-Schreier polynomial modulo terms of higher value. Then use the condition stated in the previous proposition for the definition of independent defect.
The question arose whether there is a more unified and elegant definition working simultaneously for equal and mixed characteristic.

Ramification jumps

In joint work with Olivier Piltant, the idea came up to classify defects through ramification jumps. Classically, for discretely valued fields, ramification jumps are natural numbers. However, in the case of general valuations, they can be substituted by cuts, or equivalently, final segments in the value group.

Ramification jumps

Take a valued field (K, v). Assume that $L|K$ is a Galois extension, and let $G = \mathrm{Gal}\,(L|K)$ denote its Galois group. For proper ideals I of $\mathcal{O}_L$ we consider the (upper series of) higher ramification groups

$$G_I \;:=\; \left\{ \sigma \in G \;\middle|\; \frac{\sigma b - b}{b} \in I \;\text{ for all } b \in L^\times \right\} \tag{6}$$

(see §12 of [ZS 1958]). Note that $G_{\mathcal{M}_L}$ is the ramification group of $(L|K, v)$. For every ideal I of $\mathcal{O}_L$, G_I is a normal subgroup of G (see (d) on p. 79 of [ZS 1958]). The function

$$\varphi : \; I \;\mapsto\; G_I \tag{7}$$

preserves $\subseteq$, that is, if $I \subseteq J$, then $G_I \subseteq G_J$.

Ramification jumps

As $\mathcal{O}_L$ is a valuation ring, the set of its ideals is linearly ordered by inclusion. This shows that also the higher ramification groups are linearly ordered by inclusion. Note that in general, φ will neither be injective nor surjective as a function to the set of normal subgroups of G.

We define $vL^{>0} := \{\alpha \in vL \mid \alpha > 0\}$. The function

$$v:\; I \;\mapsto\; \Sigma_I \;:=\; \{vb \mid 0 \neq b \in I\} \tag{8}$$

is an order preserving bijection from the set of all proper ideals of $\mathcal{O}_L$ onto the set of all final segments of $vL^{>0}$ (including the final segment $\emptyset$). The set of these final segments is again linearly ordered by inclusion, and the function (8) is order preserving: $J \subseteq I$ holds if and only if $\Sigma_J \subseteq \Sigma_I$ holds. The inverse of the above function is the order preserving function

$$\Sigma \;\mapsto\; I_\Sigma \;:=\; (a \in L \mid va \in \Sigma) \;=\; \{a \in L \mid va \in \Sigma\} \cup \{0\}\,. \tag{9}$$

Ramification jumps

Using the function (9), the higher ramification groups can be represented as

$$G_\Sigma \;:=\; G_{I_\Sigma} \;=\; \left\{\sigma \in G \;\middle|\; v\,\frac{\sigma b - b}{b} \in \Sigma \cup \{\infty\} \;\text{ for all } b \in L^\times \right\},$$

where Σ runs through all final segments of $vL^{>0}$.
Like the function (7), also the function $\Sigma \mapsto G_\Sigma$ is in general not injective. We call Σ a ramification jump if

$$\Sigma' \subsetneq \Sigma \;\Rightarrow\; G_{\Sigma'} \subsetneq G_\Sigma\,.$$

If Σ is a ramification jump, then I_Σ is called a ramification ideal. It follows from the definition that an ideal I of $\mathcal{O}_L$ is a ramification ideal if and only if for some subgroup G' of G, I is the smallest ideal such that $G' = G_I$ (cf. [E 1972]).

Ramification jumps

We are particularly interested in the case where $(L|K,v)$ is a Galois extension of prime degree p. Then $G = \mathrm{Gal}\,(L|K)$ is a cyclic group of order p and thus has only one proper subgroup, namely $\{\mathrm{id}\}$, and this subgroup is equal to G_Σ for $\Sigma = \emptyset$. If in this case G itself is the ramification group of the extension, then there must be a unique ramification jump Σ, and we will call I_Σ the ramification ideal of $(L|K,v)$. This ramification jump carries important information about the extension $(L|K,v)$, and it can be used to classify the defect.

Ramification jumps

Take a Galois defect extension $\mathcal{E} = (L|K, v)$ of degree $p = \operatorname{char} Kv$. For every σ in its Galois group $\operatorname{Gal}(L|K)$, with $\sigma \neq \mathrm{id}$, we set

$$\Sigma_\sigma := \left\{ v\left(\frac{\sigma b - b}{b}\right) \,\middle|\, b \in L^\times \right\} . \tag{10}$$

This is a final segment of $vL = vK$ and independent of the choice of σ (see Theorems 3.4 and 3.5 of [KR 2022]); we denote it by $\Sigma_{\mathcal{E}}$.

Theorem ([KR 2022])

$\Sigma_{\mathcal{E}}$ is the unique ramification jump of $\mathcal{E}$ and $I_{\mathcal{E}} := (a \in L \mid va \in \Sigma_{\mathcal{E}})$ is the unique ramification ideal of $\mathcal{E}$.

Classification of the defect

Here is the classification that works in both mixed characteristic case $(0,p)$ and equal characteristic case (p,p). We say that $\mathcal{E}$ has independent defect if

$$\begin{aligned}\Sigma_{\mathcal{E}} \;=\; & \{\alpha \in vK \mid \alpha > H_{\mathcal{E}}\} \text{ for some proper convex subgroup } H_{\mathcal{E}} \\ & \text{of } vK \text{ such that } vK/H_{\mathcal{E}} \text{ has no smallest positive element;}\end{aligned} \tag{11}$$

otherwise we say that $\mathcal{E}$ has dependent defect. If vK is archimedean (i.e., order isomorphic to a subgroup of $\mathbb{R}$), then condition (11) just means that $\Sigma_{\mathcal{E}}$ consists of all positive elements in vK. This definition is compatible with the definition we have already given for valued fields of positive characteristic.

Classification of the defect

Theorem

Take a Galois defect extension $\mathcal{E} = (L|K, v)$ of prime degree p. Then the following assertions are equivalent:

a) $\mathcal{E}$ has independent defect,

b) the ramification jump of $\mathcal{E}$ is equal to $\{\alpha \in vK \mid \alpha > H_{\mathcal{E}}\}$ for some proper convex subgroup $H_{\mathcal{E}}$ of vL such that $vL/H_{\mathcal{E}}$ does not have a smallest element,

c) $I_{\mathcal{E}}^{p} = I_{\mathcal{E}}$,

d) $\Omega_{\mathcal{O}_L|\mathcal{O}_K} = 0$,

e) the trace $\mathrm{Tr}_{L|K}(\mathcal{M}_L)$ is a valuation ideal $\mathcal{M}_{v_H}$ of $\mathcal{O}_K$ for some proper convex subgroup H of vK such that vK/H has no smallest positive element.

Here, $\Omega_{\mathcal{O}_L|\mathcal{O}_K}$ denotes the module of relative differentials of the ring extension $\mathcal{O}_L|\mathcal{O}_K$.

The equivalence of a) and b) was the content of the previous theorem; the remaining equivalences are proved in [CKR ?].

If assertion e) holds, then $H = H_{\mathcal{E}}$, and the valuation ring of v_H is the localization of $\mathcal{O}_L$ with respect to the ramification ideal $I_{\mathcal{E}}$. The value group of the corresponding coarsening of v does not have a smallest positive element.

Denote by $(vK)_{vp}$ the smallest convex subgroup of vK that contains vp. If vK is archimedean, or more generally, if $(vK)_{vp}$ is archimedean, then $H_{\mathcal{E}}$ can only be equal to $\{0\}$ and if the ramification ideal is a prime ideal, then it can only be equal to $\mathcal{M}_L$.

Inseparable Local Uniformization

There are indications that independent defect is more harmless than dependent defect. Michael Temkin has achieved "Inseparable Local Uniformization", i.e., local uniformization after a finite purely inseparable extension of the function field. In contrast, in [KnK 2009] local uniformization is proven after a finite separable extension of the function field. In both cases, the extension "stows the defect away".

Nevertheless, Temkin's result appears to reveal an interesting fact. Only dependent defect, the one that is connected with purely inseparable defect extensions, can be killed by purely inseparable extensions. Hence Temkin's result indicates that independent defect is more harmless and can be dealt with. Unfortunately, we have not yet succeeded to read off from Temkin's paper how this can be done with our purely valuation theoretical methods (work in progress).

Fields only admitting independent defect

Hence I started to look for valued fields beyond tame fields that would only admit independent defects. In 2016, through a talk at Oberwolfach by Bargav Bhatt on perfectoid fields and spaces it became clear to me that perfectoid fields do it. However, perfectoid fields are a too special for our purposes. By definition, they are complete and of rank one, hence they do not form an elementary class. It is better to work with deeply ramified fields. Deeply ramified extensions were first introduced by J. H. Coates and R. Greenberg in [CoGr 1996]. In [GaRa 2003], O. Gabber and L. Ramero give the following definition. A valued field (K, v) is deeply ramified if

$$\Omega_{\mathcal{O}_{K^{\mathrm{sep}}}|\mathcal{O}_K} = 0 . \qquad (12)$$

This definition does not depend on the chosen extension of the valuation from K to its separable-algebraic closure K^{sep}.

Deeply ramified fields

Then Gabber and Ramero prove that a nontrivially valued field (K, v) is deeply ramified if and only if it satisfies the following conditions:

(DRvg) if $\Gamma_1 \subsetneq \Gamma_2$ are convex subgroups of the value group vK, then Γ_2/Γ_1 is not isomorphic to $\mathbb{Z}$ (that is, no archimedean component of vK is discrete);

(DRvr) if char $Kv = p > 0$, then the homomorphism

$$\mathcal{O}_{\hat{K}}/p\mathcal{O}_{\hat{K}} \ni x \mapsto x^p \in \mathcal{O}_{\hat{K}}/p\mathcal{O}_{\hat{K}} \qquad (13)$$

is surjective, where $\mathcal{O}_{\hat{K}}$ denotes the valuation ring of the completion $\hat{K}$ of (K, v).

Axiom (DRvr) means that modulo $p\mathcal{O}_{\hat{K}}$ every element in $\mathcal{O}_{\hat{K}}$ is a p-th power.

Let us have a closer look at these conditions.

The core field in mixed characteristic

Take a valued field (K, v) of mixed characteristic. Decompose

$$v = v_0 \circ v_p \circ \overline{v}\,, \tag{14}$$

where
• v_0 is the finest coarsening of v that has residue characteristic 0,
• v_p is a rank 1 valuation on Kv_0 ,
• and $\overline{v}$ is the valuation induced by v on the residue field of v_p (which is of characteristic $p > 0$).

The valuations v_0 and $\overline{v}$ may be trivial.

In the theory of formally p-adic fields, Kv_0 is called the core field, and it is usually considered with the valuation $v_p \circ \overline{v}$.

The condition (DRvg)

From the point of view of the defect, it does not make much sense to require condition (DRvg) for archimedean classes of the value group $v_0K = vK/v_p \circ \overline{v}(Kv_0)$: since char $Kv_0 = 0$, (K, v_0) is always a defectless field (in the sense that its henselization is a defectless field), and (K, v) is a defectless field if and only if its core field is.

Following [J 2016], we call the value group vK roughly p-divisible if $v_p \circ \overline{v}\,(Kv_0)$ is p-divisible. This inspires the introduction of new elementary classes of valued fields in mixed characteristic.

Roughly tame fields

A valued field (K, v) is called a roughly tame field if it is a tame field in case it is of equal characteristic, and its core field is a tame field in case it is of mixed characteristic. Answering a question from P. Dittmann, A. Rzepka and P. Szewczyk proved in [RS ?]:

Theorem

A henselian field (K, v) is roughly tame if and only if all of its algebraic extensions are defectless fields.

In the same spirit, Y. Halevi and A. Hasson in [HH 2019] proved quantifier elimination for what one may call algebraically maximal roughly Kaplansky fields.

Roughly deeply ramified fields

We call (K, v) a roughly deeply ramified field, or in short an rdr field, if it satisfies axiom (DRvr) together with:

(DRvp) if char $Kv = p > 0$, then vp is not the smallest positive element in the value group vK.

Note that in equal characteristic, this condition is always satisfied.

Theorem

In mixed characteristic, the following assertions are equivalent:

a) (K, v) *is a roughly deeply ramified field,*

b) $(Kv_0, v_p \circ \overline{v})$ *is a deeply ramified field,*

c) (Kv_0, v_p) *is a deeply ramified field,*

d) (K, v) *satisfies (DRvr) and* vK *is roughly p-divisible.*

Semitame fields

If char $Kv = p > 0$, then (DRvg) and (DRvp) certainly hold whenever vK is divisible by p. We call (K, v) a semitame field if it satisfies condition (DRvr) together with:

(DRst) if char $Kv = p > 0$, then the value group vK is p-divisible.

We note:

Proposition

The properties (DRvg), (DRvp) and (DRst) are first order axiomatizable in the language of valued fields, and so are the classes of semitame, deeply ramified and rdr fields of fixed characteristic.

Basic facts

Theorem

1) If (K, v) is a nontrivially valued field with char $Kv = p > 0$*, then the following logical relations between its properties hold:*

tame field ⇒ separably tame field ⇒ semitame field ⇒ deeply ramified field ⇒ rdr field.

2) For a valued field (K, v) of rank 1 with char $Kv = p > 0$*, the three properties "semitame field", "deeply ramified field" and "rdr field" are equivalent.*

3) For a valued field (K, v) of mixed characteristic, condition (DRvr) holds if and only if

$$\mathcal{O}_K / p\mathcal{O}_K \ni x \mapsto x^p \in \mathcal{O}_K / p\mathcal{O}_K \tag{15}$$

is surjective.

Basic facts

Theorem

1) For a nontrivially valued field (K,v) of characteristic $p>0$, the following properties are equivalent:
a) (K,v) is a semitame field,
b) (K,v) is a deeply ramified field,
c) (K,v) is an rdr field,
d) (K,v) satisfies axiom (DRvr),
e) the completion of (K,v) is perfect,
f) (K,v) is dense in its perfect hull,
g) (K^p,v) is dense in (K,v).

2) Every perfect valued field of positive characteristic is a semitame field.

The perfect hull of $\mathbb{F}_p((t))$

By the previous theorem, $(\mathbb{F}_p((t))^{1/p^\infty}, v_t)$ is a henselian deeply ramified field. Therefore, in order to study its elementary properties, one possible approach is to study the model theory of semitame, deeply ramified and rdr fields.

Independent defect

The following theorem shows that rdr, deeply ramified and semitame fields are what we were looking for:

Theorem

All Galois defect extensions of prime degree of rdr fields have independent defect.

Algebraic extensions

The following results are proven in [GaRa 2003] and reproven by different means (not using modules of relative differentials) in [KR 2022].

Algebraic ascent:

Theorem

Every algebraic extension of a deeply ramified field is again deeply ramified. The same holds for "rdr" and "semitame" in place of "deeply ramified".

Finite descent:

Theorem

Take a finite extension $(L|K, v)$. If (L, v) is a deeply ramified field, then so is (K, v). The same holds for "rdr" and "semitame" in place of "deeply ramified".

Algebraic extensions

We see that like the classes of henselian fields and of tame fields, the class of rdr fields is closed under algebraic extensions. We obtain:

Corollary

If (K, v) is an rdr field, then for all algebraic extensions $(L|K, v)$, all Galois defect extensions of prime degree of (L, v) have independent defect.

It is an open problem whether, in analogy to the case of roughly tame fields, also the converse holds.

Algebraic extensions

For tame extensions, also infinite algebraic descent holds:

Theorem

Take a valued field (K,v), fix any extension of v to $\tilde{K}$, and let (K^r,v) be the corresponding absolute ramification field of (K,v). Then (K^r,v) is an rdr field if and only if (K,v) is, and (K^r,v) is a semitame field if and only if (K,v) is.

If (K,v) is an rdr field, then (K^r,v) is a deeply ramified field.

Algebraic extensions

Corollary

1) Take an algebraic (not necessarily finite) extension $(L|K,v)$ of valued fields. If $K^r = L^r$ with respect to some extension of v from L to $\tilde{L}$, then (L,v) is an rdr field if and only if (K,v) is, and the same holds for "semitame" in place of "rdr".

2) Take a valued field (K,v), fix any extension of v to $\tilde{K}$, and let (K^h,v) be the henselization of (K,v) in $(\tilde{K},v)$. Then (K^h,v) is a deeply ramified field if and only if (K,v) is, and the same holds for "rdr" and "semitame" in place of "deeply ramified".

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More detailed information

A lecture series on valued function fields and the defect can be found on the web page

https://math.usask.ca/fvk/Fvkls.html.

Preprints and further information:

The Valuation Theory Home Page
http://math.usask.ca/fvk/Valth.html.