Title:

Recovering the Elliptic Operator in a Reaction-diffusion equation.

Abstract:

The classical reaction-diffusion equation takes the form $u_t + Lu = f(u)$ where $L = -\nabla a(x)\nabla + q(x)$ is elliptic. This is the core equation for applications as widely separated as population dynamics and combustion theory. The usual situation is to assume that the coefficients in L, a and q, are known as is the reaction f. From this, assuming standard initial/boundary conditions, the solution $u(x,t)$ can be obtained for all later times. One inverse problem here is when some of a, q, f are unknown and we are given additional data measurements from which we seek their recovery. These measurements could be the time trace of the solution at a fixed point on the boundary or a spatial measurement at a later time T (census data).

We investigate the case when both coefficients a and q are unknown and we do so under a more general diffusion model, namely through a subdiffusion operator $\partial_t^\alpha + Lu = f(u)$, $0 < \alpha < 1$. One topic of interest is how the ill-conditioning of the inverse problem depends on the degree of subdiffusion, that is the value of α