ESI Erwin Schrödinger International Institute for Mathematics and Physics

universität wien

DVR 0065528

Lecture Series

David Jordan, U Edinburgh

Quantum character varieties in topology and representation theory

Monday, November 6, 2017 at 11:15 h
Tuesday, November 7, 2017 at 11:15 h
Wednesday, November 8, 2017 at 14:15 h
Thursday, November 9, 2017 at 11:15 h

ESI, Boltzmann Lecture Hall

Abstract: In this lecture series, I will give an overview of the construction of a fully extended topological field theory (TFT) which we call the "quantum character theory", covering various works joint with David Ben-Zvi, Adrien Brochier, Sam Gunningham, Noah Snyder, and Monica Vazirani. The lectures will be *aimed at PhD students and non-experts* so I will not assume any familiarity with the topics to be discussed. The main tools we use in the construction are quantum groups, factorization homology, and categorified representation theory, all of which we'll review in detail during the first two lectures, along with some basic notions from TFT.
In more detail: the character variety Ch_G(M) is a moduli space of representations of the fundamental group of a manifold M into a reductive group G. Character varieties of surfaces and 3-manifolds carry canonical symplectic and Lagrangian structures first introduced by Atiyah-Bott and Goldman. As the name suggests, quantum character varieties quantize these structures functorially and uniformly for all surfaces and 3-manifolds, hence giving rise to a TFT.
Once we've arrived at the theory (by Lecture 3, say), we can start recognizing a number of celebrities living in the neighborhood. I will explain relations between quantum character theory and: Witten-Reshetikhin-Turaev's 3D theory, Crane-Yetter-Kauffman's 4D theory, skein theory, and quantum A-polynomials. Outputs of the theory include reflection equation algebras, quantum differential operator algebras, Alekseev-Schomerus moduli algebras, and double affine Hecke algebras.
Finally, I will discuss applications to the development of "character sheaves" for quantum groups. Harish-Chandra introduced character sheaves to study characters of infinite-dimensional unitary representations of semi-simple Lie groups, where the naive notion of character breaks down. It turns out that the correct home for character sheaves for quantum groups is precisely the quantum character variety of the torus $T\hat{2}$. I'll discuss some starts at "Springer theory" for quantum groups coming from this perspective.

N. Carqueville

September 25, 2017