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FAKULTÄT FÜR MATHEMATIK
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Einladung zur öffentlichen Defensio von

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Thema der Dissertation:

Rhombus tilings and electrostatics

Abstract:
For over one hundred years a great many mathematicians have studied rhombus tilings of hexagons in one form or another. The story begins with Major Percy Alexander MacMahon in 1916 who was interested in objects known as boxed plane partitions (these are equivalent to rhombus tilings), and since then beautiful enumerative formulas corresponding to a total of ten different symmetry classes of tilings of hexagons have been established. The methods by which these results were derived have varied over the years, however more recently the tools and techniques used to approach such counting problems have often relied on some wonderful bijections between tilings and other families of combinatorial objects such as non-intersecting lattice paths and dimer coverings (or perfect matchings). It is perhaps safe to say that with respect to the field of combinatorics this area is well-studied and, by now, considered classical.
Arguably less well-understood is the effect that punctures in the interior of a hexagon have on the ways in which such a holey region can be tiled. For the past twenty years or so various mathematicians have embarked on quests to uncover formulas that count tilings of regions containing holes, often employing combinatorial techniques arising from the bijections described above. This has resulted in the discovery of a wide range of seemingly disparate formulas, each one pertaining to a particular family of holes.
Although a completely general formula has yet to reveal itself, many of those that have already been found exhibit, asymptotically speaking, some fascinating behaviour. It appears to be the case that on a large scale, when the holes are fixed and lie far apart from each other within tilings of the entire plane, the effect that the holes have on each other with respect to the tiles that surround them is governed by precisely the same physical principle that determines the total electric potential energy between an ensemble of static, electrically charged point particles in two dimensions. In other words, the interaction between the holes in the discrete rhombus tiling model is apparently governed by a two dimensional version of Coulombs law for electrostatics. This relationship, which places rhombus tilings of regions that contain holes firmly at the boundary between pure mathematics and statistical physics, was first conjectured by Mihai Ciucu in 2008. Almost a decade later, despite having been confirmed for a number of different families of holes, Ciucus conjecture ultimately remains wide open.
This thesis is a collection of my own investigations into this area over the past three and a half years which resulted in the discovery of a number of exact formulas that count tilings of regions containing different families of holes, found using a variety of established combinatorial methods coupled with a great deal of computational experimentation and guesswork. Where possible these formulas have been analysed asymptotically and in each case confirm Ciucus original conjecture. In some instances closely related and well-known physical phenomena chiefly, the method of images emerge directly from the mathematics, providing yet more evidence that rhombus tilings with holes discretely model two dimensional electrostatics very well indeed.

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