Gradient-polyconvex materials

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Modern approaches to elasticity are based on the assumption that the first Piola-Kirchhoff stress tensor possesses a potential called stored energy density, W, which depends on the deformation gradient. Such materials are then called hyperelastic. If we additionally assume that external forces applied on a body are conservative, equilibrium equations of elasticity are formally Euler-Lagrange equations for minimizers of the elastic-energy functional. Existence of minimizers can be ensured if W is polyconvex, for instance. Polyconvexity also allows for physically realistic behavior of W, i.e., orientation-preservation of deformations and that $W(F) \to +\infty$ if $\det F \to 0$. Nevertheless, many materials cannot obey polyconvex stored energy density. A prominent example are e.g. shape-memory alloys. A possible solution, often found in literature, is to assume that the stored energy density depends also on the second deformation gradient and is convex in it.

We show the existence of minimizers under weaker assumptions, namely, we make the energy density depend on gradients of nonlinear minors of the deformation gradients. Moreover, we outline some interesting properties of minimizers and a few applications to modeling of shape memory materials and plasticity.

This talk is based on a joint work with B. Benešová and A. Schlömerkemper (both from Würzburg).