

FAKULTÄT FÜR MATHEMATIK Dekan Univ.-Prof. Dr. Christian Krattenthaler

## Einladung zur öffentlichen Defensio von

## Sulzgruber Robin MSc

Thema der Dissertation:

## Cayley-Catalan Combinatorics of Affine Permutation Groups

Abstract: Cayley-Catalan combinatorics refers to the study of combinatorial objects counted by (generalised) Catalan numbers or Cayley numbers. Examples of classical combinatorial objects treated in this thesis that fall into this category are Dyck paths, parking functions and core partitions. These objects turn out to be closely related to the group of affine permutations and their inversions. Many involved ideas carry over to arbitrary affine Weyl groups. Exploring this connection we review the finite torus, the Shi arrangement and non-nesting parking functions. In particular, we define new combinatorial models for these objects in terms of labelled lattice paths when the crystallographic root system is of classical type.

Several combinatorial statistics on Catalan objects have been introduced to give combinatorial interpretations for polynomials appearing in representation theory or algebraic geometry. For example, Haglund's bounce-statistic, Haiman's dinv-statistic or Armstrong's skew-length of a partition have all been used to define q-analogues of Catalan numbers. We strengthen and expand on previously known symmetry properties of the skew-length statistic. The dinv-statistic is generalised to a statistic on the finite torus, allowing for a new definition of q-Catalan numbers for arbitrary Weyl groups. Furthermore, we extend the notion of Shi tableaux to give a generalisation of the skew-length statistic for affine Weyl groups, thereby enabling us to give a combinatorial definition of rational q-Catalan numbers for Weyl groups.

An important bijection in this field is the so called zeta map. The original zeta map is a bijection on the set of Dyck paths, however, it can be generalised to a uniform bijection attached to any Weyl group. We prove that this bijection transforms the dinv-statistic on elements of the finite torus into the area-statistic on non-nesting parking functions. Furthermore, we develop the lattice path combinatorics of the zeta map for the infinite families of crystallographic root systems in analogy to the connection to Dyck paths when the Weyl group is the symmetric group. This leads to the discovery of two new bijections between ballot paths and lattice paths in a square, both of which are known to be counted by central binomial coefficients.

Prüfungssenat:

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