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FAKULTÄT FÜR MATHEMATIK
Dekan Univ.-Prof. Dr. Christian Krattenthaler

Einladung zur öffentlichen Defensio von

Mag.rer.nat. Beil Sabine

Thema der Dissertation:

Triangular fully packed loop configurations: Wieland drift, configurations of small excess and a generalisation

Abstract:
In my thesis I study fully packed loop configurations (FPLs), that is, subgraphs of an $n \times n$ square grid with $4n$ external edges such that each vertex is of degree 2. They are in one-to-one correspondence with the objects of a popular model of statistical mechanics the six-vertex model and with equally famous objects in combinatorics the alternating sign matrices. Most recently, an intriguing link between FPLs and the ground state vector for a certain XXZ spin chain model and the dense $\mathcal{O}(1)$ loop model was established.
In contrast to six-vertex configurations and alternating sign matrices, (square) FPLs allow an enumerative study with respect to the connectivity of the occupied external edges. It was shown in 2004 that this study can be reduced to the study of triangular fully packed loop configurations (TFPLs) having given boundary conditions. The main focus of my thesis is on these configurations.
In the first part of the thesis Wieland drift an operation on TFPLs is defined. Its definition is based on the definition of an operation on FPLs Wieland gyration that was invented to show the rotational invariance of the numbers of FPLs having a fixed connectivity pattern. The most astonishing property of Wieland drift is that it is eventually periodic with period 1. A key step in the proof is to classify the TFPLs that are invariant under Wieland drift. It turns out that the invariance solely depends on the occurrence of a certain type of edges. A constraint that TFPLs must necessarily satisfy is given by the excess, which must be a non-negative integer. For TFPLs with excess 0 or 1 enumeration results are known. For instance, TFPLs with excess 0 are enumerated by Littlewood-Richardson coefficients. In the second part of the thesis TFPLs with excess 2 are considered. The main contribution is a linear expression for the number of these TFPLs with a given boundary in terms of numbers of TFPLs that are invariant under Wieland drift. This linear expression generalises the already existing enumeration results for TFPLs with excess 0 or 1. Its proof heavily uses Wieland drift.
In the last part of the thesis fully packed loop configurations in a hexagon (HFPLs) are introduced as a generalisation of TFPLs. In addition, many of the results for TFPLs are generalised to HFPLs. For instance, the excess is defined for HFPLs in such a way that it must be a non-negative integer and that HFPLs with excess 0 are enumerated by Littlewood-Richardson coefficients.

Prüfungssenat:

Univ.-Prof. Dr. Josef Hofbauer (Vorsitz)
(Universität Wien)

Assoz. Prof. Mag. Dr. Ilse Fischer, Privatdoz.
(Universität Wien)

Univ.-Prof. Dr. Christian Krattenthaler
(Universität Wien)

Prof. Dr. Fabrizio Caselli
(Universität Bologna)

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