

Some inequalities in the Banach space geometry

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We shall discuss some inequalities appearing in the geometry of Banach spaces:

(1) Clarkson-type inequalities, that is, classical Clarkson inequalities, generalized Clarkson inequalities, and random Clarkson Inequalities, etc. in connection with the notion of Rademacher type and cotype.

(2) Sharp triangle inequality in a Banach space.

(3) Some inequalities between von Neumann-Jordan and James constants.

In 1992 I visited Universitaet Wien and I presented a talk with the title, "From Clarkson's inequalities to Grothendieck inequality". In that talk I mentioned about random Clarkson inequality which was proved by A. Tonge [Math. Nachr. **131** (1987), 135-343], where an unknown absolute constant appeared. Then Prof. P. Michor raised a question about estimate of this constant, "even a rough estimate". At that time nothing was known about this. In this talk I shall give our answer.

Roughly speaking, before our approach almost all research on Clarkson's inequalities were devoted to prove them for various concrete Banach spaces. Our point of view is to characterize those Banach spaces where Clarkson type inequalities are valid.

Is the triangle inequality sharp enough? In fact, a sharper inequality is valid in any Banach (normed) space which is often powerful. The von Neumann-Jordan and James constants are most widely treated. We shall discuss relations between them.

References

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